

CORKS, EXOTIC 4-MANIFOLDS AND KNOT CONCORDANCE

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Dedicated to Professor Makoto Sakuma on the occasion of his 60th birthday

ABSTRACT. We give a method for producing framed knots which represent homeomorphic but non-diffeomorphic (Stein) 4-manifolds, using corks and satellite maps. To obtain the method, we introduce a new description of cork twists. As an application, we construct knots with the same 0-surgery which are not concordant for any orientations. This disproves the Akbulut-Kirby conjecture given in 1978.

1. INTRODUCTION

A framed link in S^3 gives a 4-manifold by attaching 2-handles to D^4 along the framed link. Such framed link presentations of 4-manifolds (together with gauge theoretical results) have many applications to low dimensional topology (cf. [4, 17, 27]). Thus it is a natural problem to find framed knots representing exotic (i.e. pairwise homeomorphic but non-diffeomorphic) 4-manifolds. In this paper we give a method for producing such framed knots. As an application, we disprove the Akbulut-Kirby conjecture on knot concordance given in 1978.

1.1. Framed knots representing exotic 4-manifolds. Not much is known about such framed knots. The first example of a pair of framed knots representing a pair of exotic 4-manifolds was constructed by Akbulut [3] (see also [5]). Kalmár and Stipsicz [19] extended this example to an infinite family of such pairs of knots. We remark that the framings of these known examples are all -1 . Furthermore, one 4-manifold of each pair admits a Stein structure, but the other does not.

To state our results, we recall some definitions. For a knot P in $S^1 \times D^2$, let $P(K)$ denote the (untwisted) satellite of a knot K in S^3 with the pattern P . We may regard P as a map from the set of knots in S^3 to itself. This map P is called a (untwisted) *satellite map*. For an integer n , identifying the tubular neighborhood of K with $S^1 \times D^2$ by n -framing, we have an n -*twisted satellite* $P_n(K)$ of K with the pattern P .

In this paper we give pairs of satellite maps which produce framed knots representing exotic (Stein) 4-manifolds. For the definitions of the symbols, see Section 2.

Theorem 1.1. *There exists a pair of satellite maps P and Q satisfying the following: for any fixed integer n , if a knot K in S^3 satisfies $2g_4(K) = \overline{ad}(K) + 2$ and $n \leq \widehat{tb}(K)$, then the n -twisted satellite knots $P_n(K)$ and $Q_n(K)$ with n -framings represent 4-manifolds which are homeomorphic but non-diffeomorphic to each other. Furthermore, both of these 4-manifolds admit Stein structures if $n \leq \overline{tb}(K) - 1$.*

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In fact, we construct infinitely many such pairs of satellite maps (see Theorem 4.1). They are probably mutually distinct pairs, but we do not pursue this point here. For the background of exotic Stein 4-manifolds, we refer to [8] and the references therein.

To obtain the framed knots, we introduce a new description of cork twists, which we call a *hook surgery* (see Sections 3 and 5), using a new family of corks. The hook surgery description of a cork twist is of independent interest and a key of this paper. This description immediately gives us a pair of satellite maps such that an exchange of the maps has an effect of the cork twist. Applying the construction of exotic (Stein) 4-manifolds obtained by Akbulut and the author in [7, 8], we then obtain the theorem above.

We remark that, for each integer n , there are many knots K satisfying the assumption of this theorem (e.g. positive torus knots). Furthermore, if a knot K satisfies the assumption, then $P_n(K)$ and $Q_n(K)$ also satisfy the assumption. For details, see Remark 4.2. Therefore we obtain the corollary below. We also give other types of framed knots regarding Stein structures (see Corollaries 4.10 and 4.11).

Corollary 1.2. *For each integer n , there exist infinitely many distinct pairs of n -framed knots such that each pair represents a pair of homeomorphic but non-diffeomorphic Stein 4-manifolds.*

We further discuss satellite maps from the viewpoint of 4-manifolds. For a (untwisted) satellite map P , an integer n , and a knot K , let $P^{(n)}(K)$ denote the 4-manifold represented by the (untwisted) satellite knot $P(K)$ with n -framing. We may regard $P^{(n)}$ as a map from the set of knots in S^3 to the set of smooth 4-manifolds. We use the following terminologies.

Definition 1.3. We call the map $P^{(n)}$ an *n -framed 4-dimensional satellite map*. We say that two n -framed 4-dimensional satellite maps $P^{(n)}$ and $Q^{(n)}$ are smoothly (resp. topologically) the same, if two 4-manifolds $P^{(n)}(K)$ and $Q^{(n)}(K)$ are diffeomorphic (resp. homeomorphic) to each other for any knot K in S^3 .

Our satellite maps reveal the following new difference between topological and smooth categories in 4-dimensional topology.

Corollary 1.4. *For each integer n , there exist n -framed 4-dimensional satellite maps which are topologically the same but smoothly distinct.*

In addition to these results, we also discuss a certain surgery, which we call a *dot-zero surgery*. This surgery is a natural generalization of a cork twist along a Mazur type cork, and many known exotic 4-manifolds are essentially obtained by this surgery (cf. [4, 6, 17]). We give a sufficient condition on a link such that a dot-zero surgery induced from the link does not change the smooth structure of a 4-manifold (Proposition 5.1). Applying the new description of cork twists, we then show that the effect of a dot-zero surgery along a fixed contractible 4-manifold does depend on the choice of a link presentation of the fixed 4-manifold (see Corollary 5.3).

1.2. Application to knot concordance. Let us recall the definition and the background on knot concordance. Two oriented knots $S^1 \rightarrow S^3$ are said to be concordant if there exists a proper smooth embedding $S^1 \times [0, 1] \rightarrow S^3 \times [0, 1]$ whose restriction to $S^1 \times \{0, 1\}$ are the two knots. In 1978, Akbulut and Kirby gave the following conjecture (see Problem 1.19 in the Kirby's problem list [20]).

Conjecture 1.5 (Akbulut and Kirby). *If 0-framed surgeries on two knots give the same 3-manifold, then the knots are concordant.*

To be precise, this conjecture is stated as follows. Note that there are oriented knots which are not concordant to the same knots with the reverse orientation (e.g. [21]).

Conjecture 1.6 (cf. [1]). *If 0-framed surgeries on two knots give the same 3-manifold, then the knots with relevant orientations are concordant.*

We note that many concordance invariants are defined via the 0-surgery of a knot (often with the positive meridian), and two knots with the same 0-surgery have the same concordance invariants for relevant orientations (see Problem 1.19 in [20] and introduction in [9]). Furthermore, if every homotopy 4-sphere is diffeomorphic to the standard one, then this conjecture is true when one knot is slice (see [20]). On the other hand, Cochran, Franklin, Hedden, and Horn [11] produced non-concordant pairs of knots under the weaker assumption that 0-framed surgeries on knots are homology cobordant. For related results in the link case, we refer to [9] and the references therein. Recently Abe and Tagami [1] proved that, if the slice-ribbon conjecture is true, then the Akbulut-Kirby conjecture is false.

In this paper we disprove the Akbulut-Kirby conjecture. In fact, we show that 0-framed knots given by Theorem 1.1 are counterexamples.

Theorem 1.7. *There exists a pair of knots in S^3 with the same 0-surgery which are not concordant for any orientations. Furthermore, there exist infinitely many distinct pairs of knots satisfying this condition.*

It is natural to ask whether a knot concordance invariant provides an invariant of 3-manifolds given by 0-surgeries of knots. Cochran, Franklin, Hedden, and Horn [11] proved that the invariants τ and s ([29, 32]) are not invariants of homology cobordism classes of such 3-manifolds. Our examples show

Corollary 1.8. *The knot concordance invariants τ and s are not invariants of 3-manifolds given by 0-surgeries of knots.*

2. PRELIMINARIES AND NOTATIONS

In this section, we recall basic definitions and facts and introduce our notations. Our main tools are 4-dimensional (Stein) handlebodies and Legendrian knots. For their details, we refer to [4, 17, 27].

2.1. Knots. Let K be a knot in S^3 . The *4-genus* $g_4(K)$ is defined to be the minimal genus of a properly embedded smooth surface in D^4 bounded by the knot K . For an integer n , the 4-manifold represented by K with n -framing means the 4-manifold obtained from D^4 by attaching a 2-handle along K with n -framing. The *n -shake genus* $g_s^{(n)}(K)$ is defined to be the minimal genus of a smoothly embedded closed surface representing a generator of the second homology group of this 4-manifold ([2, 4]). Note the obvious relation $g_s^{(n)}(K) \leq g_4(K)$. The lemma below is well-known and obvious from the definition.

Lemma 2.1. *Two concordant knots in S^3 have the same 4-genus.*

For an integer n , let $f_n : S^1 \times D^2 \rightarrow N(K) \subset S^3$ be a trivialization of the tubular neighborhood $N(K)$ of a knot K corresponding to the n -framing of K . For a knot P in the solid torus $S^1 \times D^2$, we denote the knots $f_0(P)$ in S^3 by $P(K)$. The knot $P(K)$ is called the *satellite* of K with the pattern P , and the knot $f_n(K)$ in S^3 is called an n -*twisted satellite* of K with the pattern P . The map on the set of knots in S^3 given by $K \mapsto P(K)$ is called a (untwisted) *satellite map*.

2.2. Legendrian knots. Throughout this paper, a Legendrian knot in S^3 means the one with respect to the standard contact structure on S^3 . For a Legendrian knot \mathcal{K} in S^3 , let $tb(\mathcal{K})$ and $r(\mathcal{K})$ denote the Thurston-Bennequin number and the rotation number, respectively. According to [13, 16], the 4-manifold represented by \mathcal{K} with the framing $tb(\mathcal{K}) - 1$ admits a Stein structure. Since any Stein 4-manifold can be embedded into a closed minimal complex surface of general type with $b_2^+ > 1$ ([24]), the well-known adjunction inequality for this closed 4-manifold ([14, 22, 26, 28]) together with Gompf's Chern class formula ([16]) gives the following version for the Stein 4-manifold. Note that this holds even for the genus zero case (cf. [17, 27, 8]), unlike the version for general closed 4-manifolds.

Theorem 2.2 ([5, 25]). $tb(\mathcal{K}) - 1 + |r(\mathcal{K})| \leq 2g_s^{(tb(\mathcal{K})-1)}(\mathcal{K}) - 2$.

We define the *adjunction number* $ad(\mathcal{K})$ as the left side of this inequality, namely,

$$ad(\mathcal{K}) = tb(\mathcal{K}) - 1 + |r(\mathcal{K})|.$$

For a knot K in S^3 , we say that a Legendrian knot \mathcal{K} is a Legendrian representative of K if \mathcal{K} is smoothly isotopic to K . We denote the set of Legendrian representatives of K by $\mathcal{L}(K)$. We define the *maximal Thurston-Bennequin number* $\overline{tb}(K)$ and the *maximal adjunction number* $\overline{ad}(K)$ of K by

$$\overline{tb}(K) = \max\{tb(\mathcal{K}) \mid \mathcal{K} \in \mathcal{L}(K)\} \quad \text{and} \quad \overline{ad}(K) = \max\{ad(\mathcal{K}) \mid \mathcal{K} \in \mathcal{L}(K)\}.$$

We also define the symbol $\widehat{tb}(K)$ by

$$\widehat{tb}(K) = \max\{tb(\mathcal{K}) \mid \mathcal{K} \in \mathcal{L}(K), ad(\mathcal{K}) = \overline{ad}(K)\}.$$

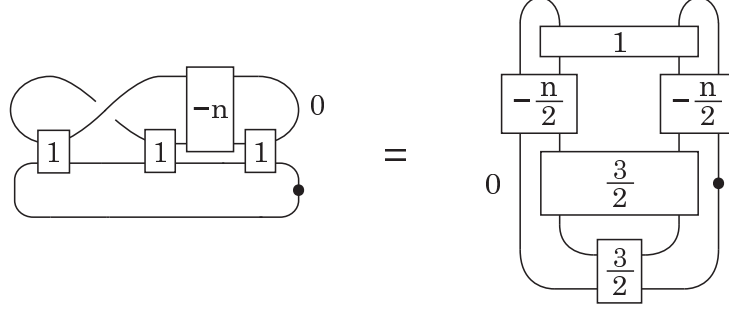
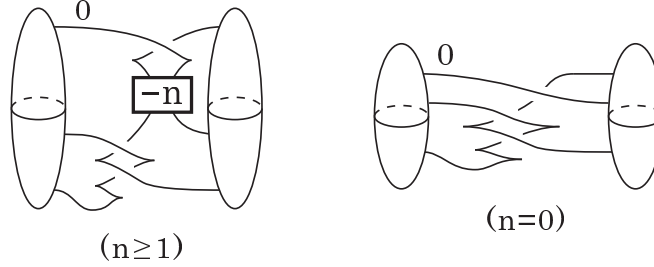
2.3. Corks. For a compact contractible smooth 4-manifold C and an involution $\tau : \partial C \rightarrow \partial C$, the pair (C, τ) is called a *cork*, if τ extends to a self-homeomorphism of C but does not extend to any self-diffeomorphism of C . (We assume that a cork is contractible for simplicity, though we did not in [6].) For a smooth 4-manifold X containing C as a submanifold, remove C from X and glue it via the involution τ . We call this operation a *cork twist* along (C, τ) . We note that any self-diffeomorphism on the boundary of a smooth contractible 4-manifold extend to a self-homeomorphism of the 4-manifold ([15]). For examples of corks, see [6].

3. A NEW DESCRIPTION OF CORK TWISTS

In this section, we give a new family of corks and introduce a new description of cork twists, which we call a hook surgery in Section 5. The new description is a key of our main results and has independent interest.

We first construct a new family of corks. For an integer n , let V_n be the contractible 4-manifold given by the left diagram of Figure 1. It is easy to check that this manifold is diffeomorphic to the right 4-manifold by isotopy, where the box $-\frac{n}{2}$ denotes $-n$ right handed half twists. Note that the link is symmetric. Let $g_n : \partial V_n \rightarrow \partial V_n$ be the involution obtained by first surgering $S^1 \times D^3$ to $D^2 \times S^2$

and then surgering the other $D^2 \times S^2$ to $S^1 \times D^3$ (i.e. replacing the dot and the zero in the diagram). In the case $n \geq 0$, by converting the 1-handle notation, we obtain the Stein handlebody presentation of V_n in Figure 2, where the left handed full twists in the box denote the Legendrian version shown in Figure 3. Hence according to [13, 16], V_n admits a Stein structure for $n \geq 0$. We remark that (V_0, g_0) is the same cork as (W_1, f_1) in [6].

FIGURE 1. Two diagrams of V_n FIGURE 2. A Stein handlebody presentation of V_n ($n \geq 0$)

We can easily show the lemma below similarly to [5, 6].

Lemma 3.1. *(V_n, g_n) is a cork for any integer $n \geq 0$.*

We next introduce a new description of cork twists. For an integer n , let V_n^* be the left contractible 4-manifold in Figure 4. Note that this manifold is diffeomorphic to the right 4-manifold by isotopy. We present a knot K in S^3 by using a tangle T_K as shown in Figure 5. We show that a cork twist along (V_n, g_n) can be obtained as in the lower side of Figure 6. Note that the upper side describes a cork twist along (V_n, g_n) . In particular, for any knot K and any integers n, k , hooking the k -framed knot K to the 1-handles of V_n and V_n^* in the same way has an effect of a cork twist along the obvious (V_n, g_n) . This is of independent interest because all known cork twists are described by an exchange of a dot and zero in the language of handlebody diagram (e.g. the upper side). Due to this description, one might expect that V_n^* is an exotic copy of V_n . However, it turns out that V_n^* is diffeomorphic to V_n .

To obtain the new description, we construct a diffeomorphism $\partial V_n \rightarrow \partial V_n^*$. For a knot K in S^3 , let γ_K and γ_K^* be knots in ∂V_n and ∂V_n^* given by Figure 7, respectively.

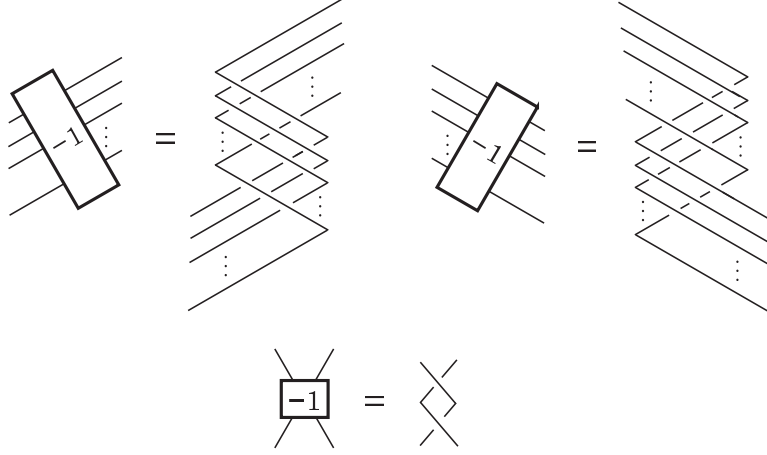
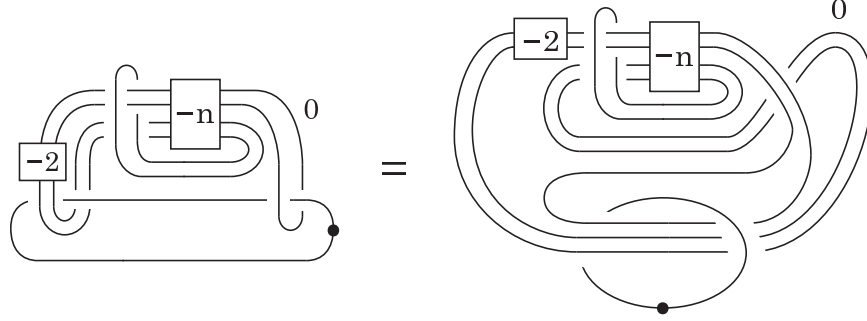
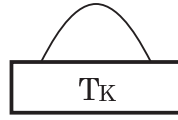


FIGURE 3. Legendrian versions of a left handed full twist

FIGURE 4. Two diagrams of V_n^* FIGURE 5. A knot K in S^3 given by a tangle T_K .

Theorem 3.2. *For each integer n , there exists a diffeomorphism $g_n^* : \partial V_n \rightarrow \partial V_n^*$ satisfying the following conditions.*

- *The diffeomorphism g_n^* sends the knot γ_K to γ_K^* for any knot K in S^3 .*
- *The diffeomorphism $g_n^* \circ g_n^{-1} : \partial V_n \rightarrow \partial V_n^*$ extends to a diffeomorphism $V_n \rightarrow V_n^*$. In particular, V_n^* is diffeomorphic to V_n .*

Proof. Let $g_n^* : \partial V_n \rightarrow \partial V_n^*$ be the diffeomorphism defined by Figure 8. (For the step from the seventh diagram to the eighth, use the isotopy in Figure 9 after canceling the 1-handle.) The first condition of the claim is obvious from the figure. The diffeomorphism $g_n^* \circ g_n^{-1}$ is clearly given by the procedure from the second diagram

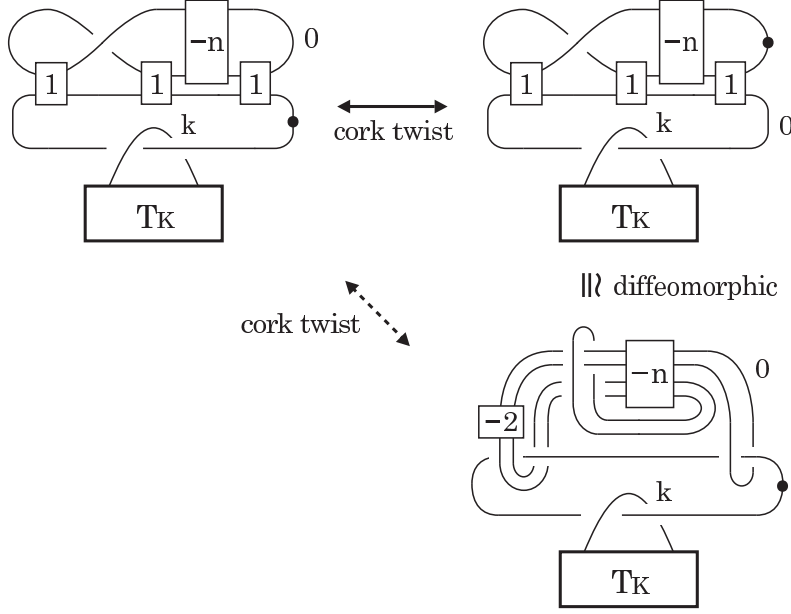
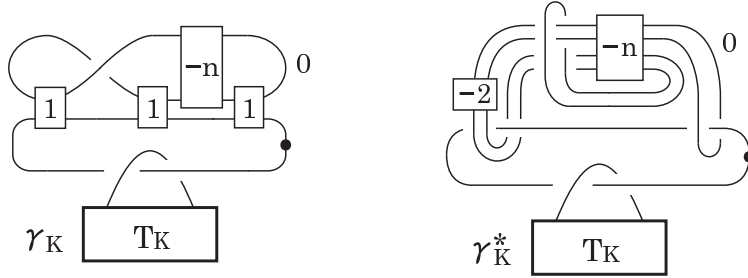


FIGURE 6. A new description of a cork twist (lower side)

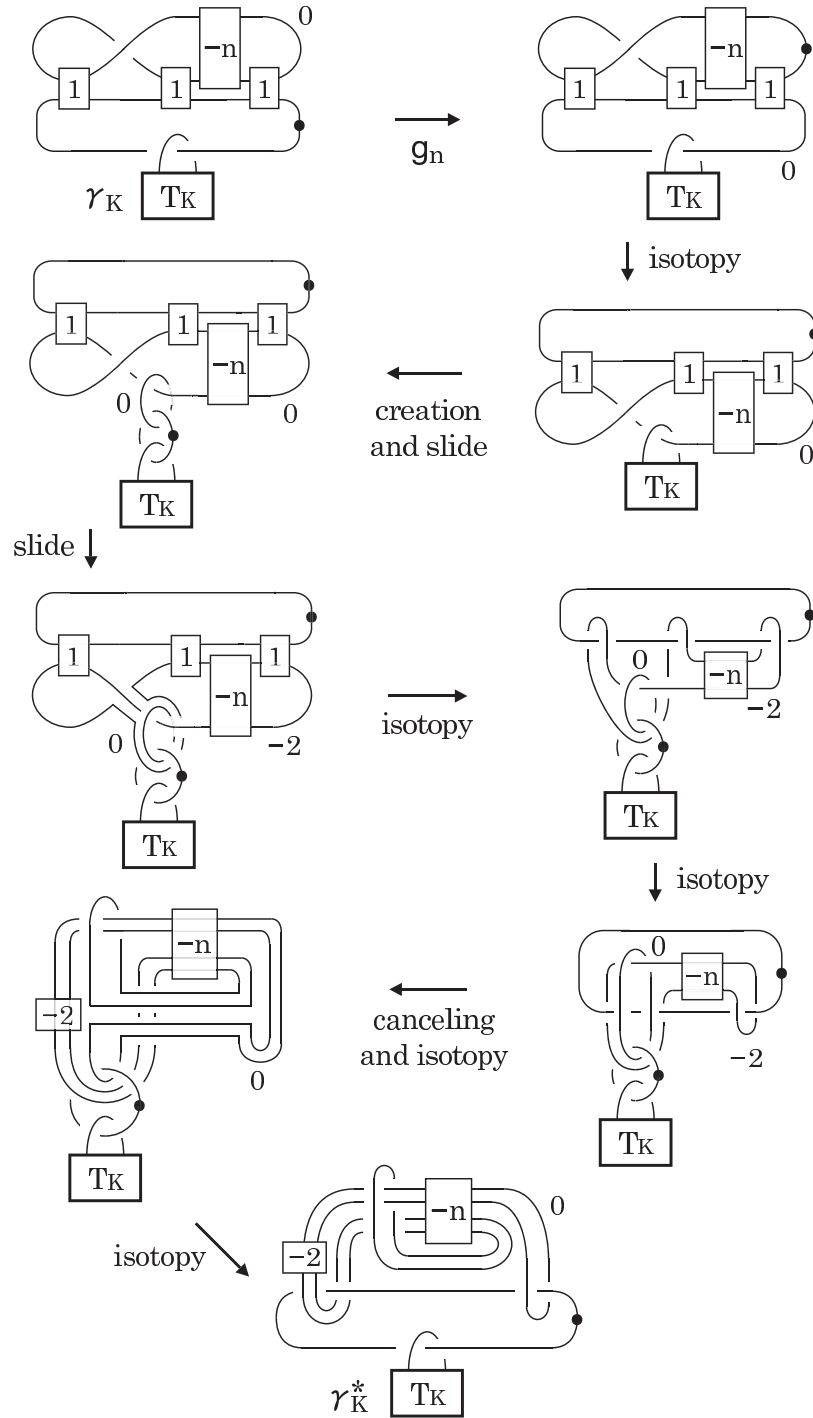
FIGURE 7. Knots γ_K and γ_K^* in ∂V_n and ∂V_n^*

to the last diagram in the figure. Since each step is induced from a diffeomorphism between the obvious 4-manifolds, the second condition follows. \square

Corollary 3.3. *Assume that a smooth 4-manifold X contains V_n ($n \geq 0$) as a submanifold. Then the 4-manifold obtained from X by removing V_n and gluing V_n^* via the gluing map g_n^* is diffeomorphic to the 4-manifold obtained from X by the cork twist along (V_n, g_n) .*

Proof. Since $g_n^* \circ g_n^{-1} : \partial V_n \rightarrow \partial V_n^*$ extends to a diffeomorphism $V_n \rightarrow V_n^*$, and $g_n^* = (g_n^* \circ g_n^{-1}) \circ g_n$, the claim follows. \square

In summary, V_n^* is diffeomorphic to V_n , and a cork twist along (V_n, g_n) has the same effect as a surgery along V_n via the gluing map $g_n^* : \partial V_n \rightarrow \partial V_n^*$, as shown in Figure 6.

FIGURE 8. The diffeomorphism $g_n^* : \partial V_n \rightarrow \partial V_n^*$

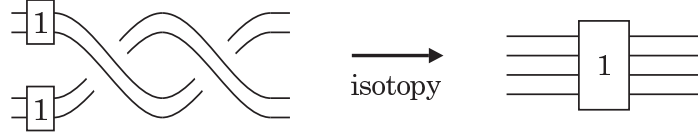
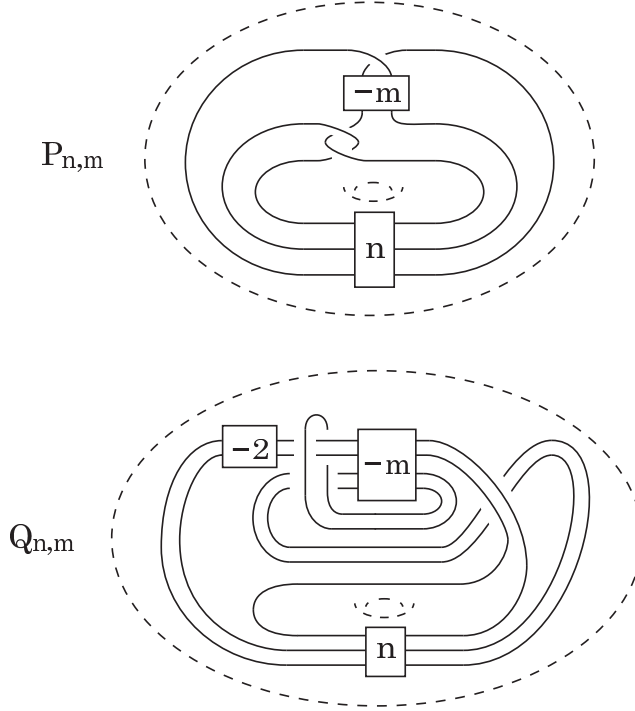


FIGURE 9. Isotopy

4. PROOFS OF THE MAIN RESULTS

In this section we prove our main results. For integers n, m , let $P_{n,m}$ and $Q_{n,m}$ be the knots in unknotted solid tori in S^3 given by Figure 10. Here the dotted lines indicate solid tori, and the box n denotes $|n|$ right (resp. left) handed full twists if n is positive (resp. negative). In the case where $m = 0$, they have the simple diagrams shown in Figure 11. We regard $P_{n,m}$ and $Q_{n,m}$ as (untwisted) satellite maps. Note that the knot $P_{n,m}(K)$ (resp. $Q_{n,m}(K)$) is isotopic to the n -twisted satellite of a knot K with the pattern $P_{0,m}$ (resp. $Q_{0,m}$). Let $P_{n,m}^{(n)}(K)$ (resp. $Q_{n,m}^{(n)}(K)$) denote the 4-manifold represented by the knot $P_{n,m}(K)$ (resp. $Q_{n,m}(K)$) with n -framing.

FIGURE 10. Pattern knots $P_{n,m}$ and $Q_{n,m}$ ($n, m \in \mathbb{Z}$) in unknotted solid tori in S^3 .

Here we prove that the pair of satellite maps $P_{n,m}$ and $Q_{n,m}$ produces many pairs of n -framed non-concordant knots, each pair of which represents a pair of exotic (Stein) 4-manifolds.

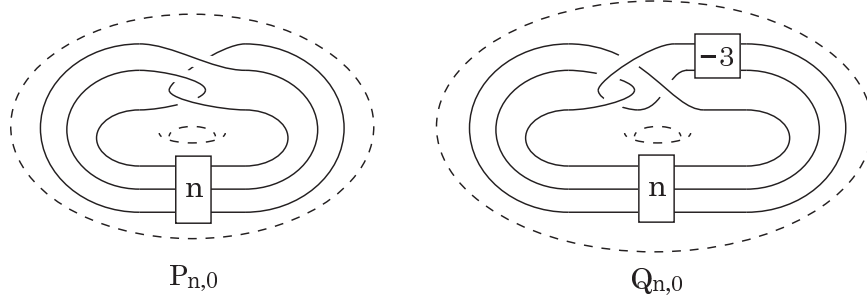


FIGURE 11. Pattern knots $P_{n,0}$ and $Q_{n,0}$ ($n \in \mathbb{Z}$) in unknotted solid tori in S^3 .

Theorem 4.1. *Fix integers n and m with $m \geq 0$. For each knot K in S^3 satisfying $2g_4(K) = \overline{ad}(K) + 2$ and $n \leq \widehat{tb}(K)$, the knots $P_{n,m}(K)$ and $Q_{n,m}(K)$ satisfy the following conditions.*

- The 4-manifolds $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$ are homeomorphic but not diffeomorphic to each other. Furthermore, both of these 4-manifolds admit Stein structures if $n \leq \overline{tb}(K) - 1$.
- $g_4(P_{n,m}(K)) = g_4(K) + 1$ and $g_4(Q_{n,m}(K)) = g_4(K)$.

Consequently, the knots $P_{n,m}(K)$ and $Q_{n,m}(K)$ are not concordant for any orientations, and their n -surgeries yield the same 3-manifold.

Remark 4.2. (1) There are many knots satisfying $2g_4(K) = \overline{ad}(K) + 2$. For example, it is well-known that any positive (p, q) -torus knot $T_{p,q}$ satisfies this assumption, and $\widehat{tb}(T_{p,q}) = \overline{tb}(T_{p,q}) = pq - p - q$. This can be easily seen from an appropriate Legendrian realization and the adjunction inequality.

(2) If a knot K in S^3 satisfies the assumption of this theorem for fixed integers n, m , then both of the knots $P_{n,m}(K)$ and $Q_{n,m}(K)$ also satisfy the assumption (see Remark 4.5). Therefore, for each integer n , just by iterating this operation to any knot satisfying the assumption, we obtain infinitely many distinct pairs of n -framed non-concordant knots such that each pair represents a pair of exotic (Stein) 4-manifolds.

(3) The knot $Q_{n,m}(K)$ is concordant to K for any integers n, m . This can be easily seen from the diagram of $Q_{n,m}$ by checking that the pattern knot $Q_{n,m}$ is a band sum of the longitude of the solid torus and an unknot which are unlinking.

To prove this theorem, we show the lemma below. As seen from the proof, we obtained the satellite maps $P_{n,m}$ and $Q_{n,m}$ from the cork V_m and its alternative description V_m^* .

Lemma 4.3. *For any integers n, m and any knot K in S^3 , the 4-manifold $P_{n,m}^{(n)}(K)$ is homeomorphic to $Q_{n,m}^{(n)}(K)$.*

Proof. We present a knot K using a tangle T_K as in Figure 5. We can easily check that $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$ are respectively diffeomorphic to the left and the upper right 4-manifolds in Figure 12, by canceling the 1-handles (see also Figure 2 and the right diagram in Figure 4). By Theorem 3.2, $Q_{n,m}^{(n)}(K)$ is obtained from

$P_{n,m}^{(n)}(K)$ by removing V_m and gluing V_m^* via the gluing map g_n^* . Since any diffeomorphism between the boundaries of contractible smooth 4-manifolds extends to a homeomorphism between the 4-manifolds ([15]), the claim follows. \square

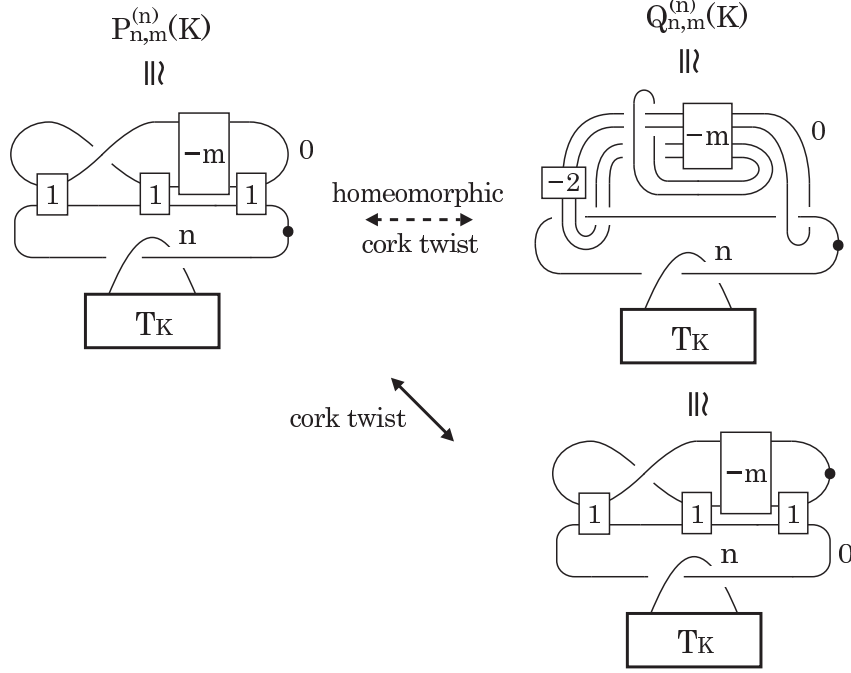


FIGURE 12. The left and the right 4-manifolds are diffeomorphic to $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$, respectively.

Remark 4.4. It follows from Corollary 3.3 (see also Figure 12) that $P_{n,m}^{(n)}(K)$ is obtained from $Q_{n,m}^{(n)}(K)$ by a cork twist along (V_m, g_m) (in the case $m \geq 0$).

Akbulut and the author [8] gave an algorithm which produces arbitrarily many exotic Stein 4-manifolds by applying corks. Adapting the argument to our simple case, we prove Theorem 4.1.

Proof of Theorem 4.1. Fix integers n and m with $m \geq 0$. Let K be a knot in S^3 satisfying $2g_4(K) = \overline{ad}(K) + 2$ and $n \leq \widehat{tb}(K)$. We first give Legendrian representatives of the satellite knots $P_{n,m}(K)$ and $Q_{n,m}(K)$. Due to the assumption on K , there exists a Legendrian representative of K with $tb = \widehat{tb}(K)$ and $ad = \overline{ad}(K)$. Since $n \leq \widehat{tb}(K)$, by adding zig-zags to a front diagram of the representative, we get a Legendrian representative \mathcal{K} of K satisfying $n = tb(\mathcal{K})$ and $ad(\mathcal{K}) = \overline{ad}(K)$. We present a front diagram of \mathcal{K} by a Legendrian tangle $\mathcal{T}_{\mathcal{K}}$ as in the left diagram of Figure 13. We then draw the front diagram consisting of three copies of \mathcal{K} each of which is slightly shifted to the vertical direction (cf. upper Legendrian pictures in Figure 3). We present the resulting front diagram by a Legendrian tangle $\mathcal{T}_{\mathcal{K}}^3$ as in the right diagram. Using this tangle, we obtain the Legendrian representatives

of $P_{n,m}(K)$ and $Q_{n,m}(K)$ in Figures 14 and 15, respectively. Here the left handed full twists in the boxes denote the Legendrian versions shown in Figure 3.



FIGURE 13. The left is a front diagram of \mathcal{K} . The right is a front diagram consisting of three copies of \mathcal{K} slightly shifted to the vertical direction.

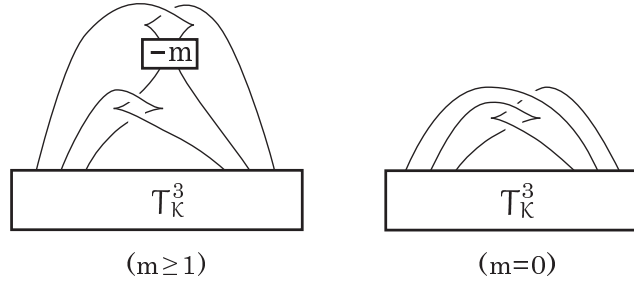


FIGURE 14. A Legendrian representative of $P_{n,m}(K)$ ($m \geq 0$)

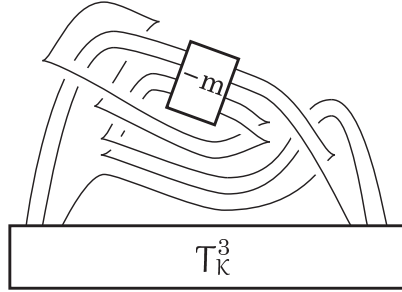


FIGURE 15. A Legendrian representative of $Q_{n,m}(K)$ ($m \geq 0$)

Next we determine the 4-genus and the n -shake genus of $P_{n,m}(K)$. By counting the writhe and the number of left cusps of the front diagram, one can easily check that the Legendrian representative of $P_{n,m}(K)$ satisfies $tb = tb(\mathcal{K}) + 2$ and $|r| = |r(\mathcal{K})|$. By adding a zig-zag to this front diagram, we get a Legendrian representative of $P_{n,m}(K)$ with $tb = n + 1$ and $|r| = |r(\mathcal{K})| + 1$. Applying the adjunction inequality, we see

$$n + |r(\mathcal{K})| + 1 \leq 2g_s^{(n)}(P_{n,m}(K)) - 2.$$

Since $n - 1 + |r(\mathcal{K})| = \overline{ad}(K)$ and $2g_4(K) = \overline{ad}(K) + 2$, this inequality implies $g_s^{(n)}(P_{n,m}(K)) \geq g_4(K) + 1$. On the other hand, we see that $P_{n,m}(K)$ becomes isotopic to \mathcal{K} after changing the lowest crossing shown in Figure 14. This implies that $P_{n,m}(K)$ bounds a surface of genus $g_4(K) + 1$ in D^4 . Therefore, we obtain

$$g_s^{(n)}(P_{n,m}(K)) = g_4(P_{n,m}(K)) = g_4(K) + 1.$$

Since $Q_{n,m}(K)$ is concordant to K (Remark 4.2), we also see

$$g_4(Q_{n,m}(K)) = g_4(K).$$

Here we distinguish smooth structures on $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$. By Lemma 4.3, the 4-manifold $P_{n,m}^{(n)}(K)$ is homeomorphic to $Q_{n,m}^{(n)}(K)$. On the other hand, the above arguments show

$$g_s^{(n)}(P_{n,m}(K)) = g_4(K) + 1 > g_4(Q_{n,m}(K)) \geq g_s^{(n)}(Q_{n,m}(K)).$$

Consequently, $g_s^{(n)}(P_{n,m}(K)) \neq g_s^{(n)}(Q_{n,m}(K))$. Therefore, it follows from the definition of the n -shake genus that $P_{n,m}^{(n)}(K)$ is not diffeomorphic to $Q_{n,m}^{(n)}(K)$.

Lastly we check existence of Stein structures on $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$. By using the left and the lower right diagrams of $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$ in Figure 12 and the Stein handlebody diagram of V_n in Figure 2, we can easily realize $P_{n,m}^{(n)}(K)$ (resp. $Q_{n,m}^{(n)}(K)$) as a Stein handlebody for $n \leq \widehat{tb}(K)$ (resp. $n \leq \widehat{tb}(K) - 1$). Hence, according to [13, 16], these 4-manifolds admit Stein structures. \square

Remark 4.5 (Knots $P_{n,m}(K)$ and $Q_{n,m}(K)$). Fix integers n, m with $m \geq 0$. For a knot K satisfying the assumption of Theorem 4.1, the knots $P_{n,m}(K)$ and $Q_{n,m}(K)$ satisfy the conditions below, as seen from the proof.

$$\begin{aligned} 2g_4(P_{n,m}(K)) &= \overline{ad}(P_{n,m}(K)) + 2, & \widehat{tb}(P_{n,m}(K)) &\geq n + 2. \\ 2g_4(Q_{n,m}(K)) &= \overline{ad}(Q_{n,m}(K)) + 2, & \widehat{tb}(Q_{n,m}(K)) &\geq n. \end{aligned}$$

Remark 4.6 (The n -shake genera). Fix integers n, m with $m \geq 0$. Assume that a knot K satisfies the assumption of Theorem 4.1. Then the n -shake genera of $P_{n,m}(K)$ and $Q_{n,m}(K)$ are given as follows.

$$\begin{aligned} g_s^{(n)}(P_{n,m}(K)) &= g_4(K) + 1, & \text{if } n \leq \widehat{tb}(K). \\ g_s^{(n)}(Q_{n,m}(K)) &= g_4(K), & \text{if } n \leq \widehat{tb}(K) - 1. \end{aligned}$$

The former equality is obvious from the proof of Theorem 4.1. The latter can be seen as follows. It is easy to realize the lower right handlebody in Figure 12 as a Stein handlebody, if $n \leq \widehat{tb}(K) - 1$. Therefore, this manifold admits a Stein structure. Applying the adjunction inequality for Stein 4-manifolds ([5, 25]. cf. [17, 27]) to this 4-manifold, we obtain the above equality.

Remark 4.7 (The knot concordance invariants τ and s). Let n, m, K be as in Remark 4.6. Then τ and s of $P_{n,m}(K)$ and $Q_{n,m}(K)$ are given as follows.

$$\begin{aligned} 2\tau(P_{n,m}(K)) &= s(P_{n,m}(K)) = 2g_4(P_{n,m}(K)) = \overline{ad}(K) + 4, \\ 2\tau(Q_{n,m}(K)) &= s(Q_{n,m}(K)) = 2g_4(Q_{n,m}(K)) = \overline{ad}(K) + 2. \end{aligned}$$

These are straightforward from Remark 4.5 and the following inequalities for an arbitrary knot L in S^3 ([30], [31], [34]. cf. [11]).

$$\overline{ad}(L) \leq 2\tau(L) - 2 \leq 2g_4(L) - 2, \quad \overline{ad}(L) \leq s(L) - 2 \leq 2g_4(L) - 2.$$

Remark 4.8. (1) The n -shake genus (and thus 4-genus) of $P_{n,m}(K)$ can be obtained also by applying the original argument of Akbulut and the author [8], that is, by applying the adjunction inequality to the (Stein version of) left handlebody in Figure 12. This method clearly works for many other satellite knots. The argument of this paper is a simplification of this method by canceling the 1-handle.

(2) Shortly after this paper appeared on arXiv, Ray kindly informed the author that the above values of \widehat{tb} , the n -shake genus, the 4-genus, τ and s for $P_{n,m}(K)$ follow from earlier (but later than [8]) results of Cochran-Ray [12], which were obtained by the similar arguments. Furthermore, they also discussed iterations of satellite operations. We remark that their notations are different from ours. See also Ray [33] for interesting applications of iterations.

We give a simple example in the case where the framing is 0.

Example 4.9. We consider the right handed trefoil knot $T_{2,3}$. This knot satisfies the assumption of Theorem 4.1 and $\widehat{tb}(T_{2,3}) = \overline{tb}(T_{2,3}) = 1$. Hence by Theorem 4.1, the knots $P_{0,0}(T_{2,3})$ and $Q_{0,0}(T_{2,3})$ are non-concordant for any orientations, and their 0-surgeries yield the same 3-manifold. Furthermore, these knots with 0-framings represent Stein 4-manifolds which are homeomorphic but non-diffeomorphic to each other. Figure 16 gives diagrams of these knots.

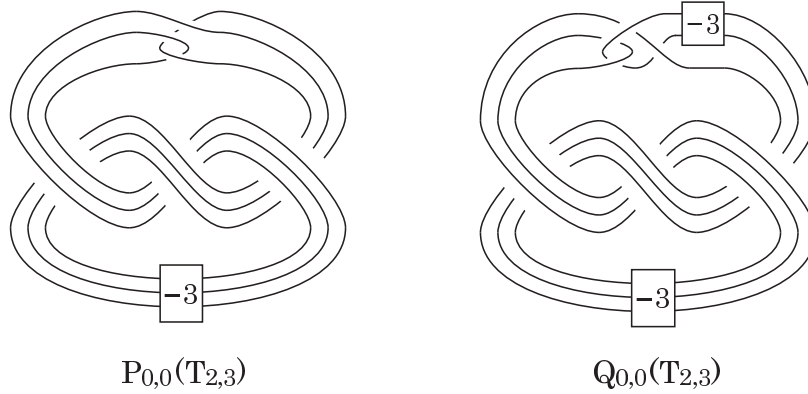


FIGURE 16. $P_{0,0}(T_{2,3})$ and $Q_{0,0}(T_{2,3})$

Now we can easily prove our main results.

Proof of Theorems 1.1 and 1.7 and Corollary 1.2. These are obvious from Theorem 4.1 and Remark 4.2. Note that we can distinguish the pairs of knots by comparing their 4-genera. \square

Proof of Corollary 1.4. This is obvious from Lemma 4.3, Theorem 4.1 and Remark 4.2. \square

Proof of Corollary 1.8. This is obvious from Theorem 4.1 and Remarks 4.2 and 4.7. \square

Regarding Stein structures, Theorem 4.1 also gives framed knots which represent exotic 4-manifolds different from those in Corollary 1.2.

Corollary 4.10. *Fix integers n, m with $m \geq 0$. Assume that a knot K satisfies $n = \overline{tb}(K) = 2g_4(K) - 1$. Then the 4-manifolds $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$ are homeomorphic but not diffeomorphic to each other. Furthermore, $P_{n,m}^{(n)}(K)$ admits a Stein structure, but $Q_{n,m}^{(n)}(K)$ does not admit any Stein structure.*

Proof. The former claim follows from Theorem 4.1. The proof of Theorem 4.1 shows that $P_{n,m}^{(n)}(K)$ admits a Stein structure. By the inequality $g_s^{(n)}(Q_{n,m}(K)) \leq g_4(Q_{n,m}(K)) = g_4(K)$ and the assumption, we see $2g_s^{(n)}(Q_{n,m}(K)) - 2 > n$. Since n is the self-intersection number of the generator of $H_2(Q_{n,m}^{(n)}(K); \mathbb{Z})$, the adjunction inequality for Stein 4-manifolds ([5, 25]. cf. [17, 27]) guarantees that $Q_{n,m}^{(n)}(K)$ does not admit any Stein structure. \square

Corollary 4.11. *For each integer n , there exist infinitely many distinct pairs of n -framed knots such that each pair represents a pair of homeomorphic but non-diffeomorphic non-Stein 4-manifolds.*

Proof. Let n, m be fixed integers with $m \geq 0$. Assume that a knot K satisfies $2g_4(K) = \overline{ad}(K) + 2$, $n \leq \widehat{tb}(K)$, and $n \leq \overline{tb}(K) - 1$. We denote the mirror images of the knots $P_{n,m}(K)$ and $Q_{n,m}(K)$ by $\overline{P}_{n,m}(K)$ and $\overline{Q}_{n,m}(K)$. We here show that the 4-manifold represented by $\overline{P}_{n,m}(K)$ with $(-n)$ -framing does not admit any Stein structure. Suppose, to the contrary, that this 4-manifold X admits a Stein structure. Then the boundary connected sum $Z := X \natural P_{n,m}^{(n)}(K)$ admits a Stein structure, since $P_{n,m}^{(n)}(K)$ admits a Stein structure. By a handle slide, we see that Z contains an embedded 2-sphere representing a non-zero second homology class with the self-intersection number 0. (Note that the connected sum $P_{n,m}(K) \# \overline{P}_{n,m}(K)$ is a ribbon knot.) This contradicts the adjunction inequality for Stein 4-manifolds ([5, 25]. cf. [17, 27]). Hence X does not admit any Stein structure. The same argument clearly works for $\overline{Q}_{n,m}(K)$. Therefore, the knots $\overline{P}_{n,m}(K)$ and $\overline{Q}_{n,m}(K)$ with $(-n)$ -framings represent non-Stein 4-manifolds. Since these non-Stein 4-manifolds are the 4-manifolds $P_{n,m}^{(n)}(K)$ and $Q_{n,m}^{(n)}(K)$ with the reverse orientations, Theorem 4.1 and Remark 4.2 imply the claim by varying K . \square

One might ask whether our counterexamples to the Akbulut-Kirby conjecture are topologically concordant. We point out that there are infinitely many topologically concordant examples among them by an argument similar to [11].

Corollary 4.12. *There exists a pair of knots with the same 0-surgery which are topologically concordant but smoothly non-concordant for any orientations. Furthermore, there exist infinitely many distinct pairs of knots satisfying this condition.*

Proof. Let $m \geq 0$ be a fixed integer, and let K be a topologically slice knot with $2g_4(K) = \overline{ad}(K) + 2$ and $0 \leq \widehat{tb}(K)$. For example, the untwisted Whitehead double of a positive torus knot satisfies this condition (e.g. [5]). Note that the Whitehead double of a knot is topologically slice as is well-known. It is known that if two knots are topologically concordant, then the images of them by a satellite map are also

topologically concordant (e.g. [10]). Hence $P_{0,m}(K)$ and $Q_{0,m}(K)$ are topologically slice knots. Therefore, by Theorem 4.1, the pair of knots $P_{0,m}(K)$ and $Q_{0,m}(K)$ satisfies the claim. Remark 4.2 thus tells that we obtain infinitely many distinct such pairs by iterating this construction. \square

Let us recall that each of our counterexamples to the Akbulut-Kirby conjecture is a pair of knots $P_{0,m}(K)$ and $Q_{0,m}(K)$ ($m \geq 0$), where K is an arbitrary knot satisfying $2g_4(K) = \overline{ad}(K) + 2$ and $0 \leq \widehat{tb}(K)$. Levine kindly pointed out to the author that, at least in the $m = 0$ case, the condition on K can be relaxed by combining a result of his paper [23], namely, the corollary below holds. Here ϵ is an invariant of knot concordance defined by Hom [18].

Corollary 4.13. *Assume that a knot K satisfies either $\tau(K) > 0$ or $\epsilon(K) = -1$. Then the knots $P_{0,0}(K)$ and $Q_{0,0}(K)$ have the same 0-surgery but are not concordant for any orientations.*

Proof. A result of Levine [23] shows $\tau(P_{0,0}(K)) = \tau(K) + 1$ under the above assumption. Hence $P_{0,0}(K)$ is not concordant to K for any orientations. Since $Q_{0,0}(K)$ is concordant to K (Remark 4.2), this fact and Lemma 4.3 shows the claim. \square

We remark that this corollary enlarges the class of K from the class given by Theorem 4.1. For example, a negative torus knot $T_{p,-q}$ ($p, q > 1$) satisfies the assumption of this corollary by [18], though it does not satisfy the aforementioned condition given by Theorem 4.1.

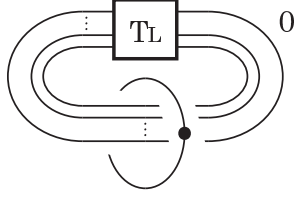
Remark 4.14 (Extension of the main construction). For simplicity, we defined the cork (V_m, g_m) and the pattern knots $P_{n,m}$ and $Q_{n,m}$ in the case where m is an integer. Clearly, we can extend these definitions to the case where m is a half-integer. Moreover, we can similarly prove our results for any positive half-integer m . We remark that $(V_{\frac{1}{2}}, g_{\frac{1}{2}})$ is the same cork as $(\overline{W}_1, \overline{f}_1)$ in [6].

5. HOOK SURGERY AND DOT-ZERO SURGERY

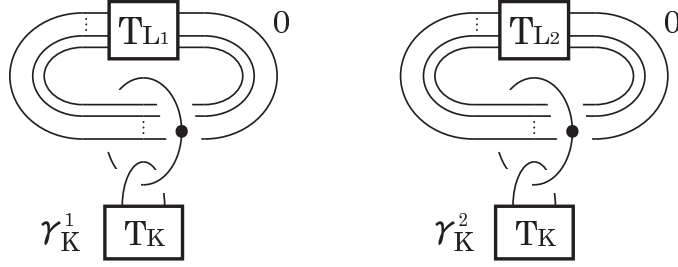
In this section we first introduce a hook surgery as a formulation of the new description of cork twists obtained in Section 3. We next discuss a certain surgery which we call a dot-zero surgery. This surgery is a generalization of cork twists along Mazur type corks. Specifically we give a sufficient condition on a link in S^3 such that any dot-zero surgery induced from the link does not change the smooth structure of a 4-manifold. Applying Stein 4-manifolds V_n and V_n^* obtained in Section 3, we then show that the effect of a dot-zero surgery along a fixed contractible 4-manifold does depend on the choice of a link presentation of the fixed 4-manifold.

5.1. Hook surgery. For a given link L of two components in S^3 with an unknotted component U , by putting a dot on U and a 0 on top of the other component, we get a handlebody diagram as shown in Figure 17. Here T_L denotes a tangle representing L . We denote the resulting 4-dimensional handlebody by X_L .

Now let L_1 and L_2 be two links satisfying the above condition of L . We further assume that the linking number of the two components of each L_i is one. Note that both X_{L_1} and X_{L_2} are contractible due to this assumption. For a knot K in S^3 , we present K as in Figure 5 using a tangle T_K . We define knots γ_K^1 and γ_K^2 in ∂X_{L_1} and ∂X_{L_2} by Figure 18. Suppose that a diffeomorphism $\varphi : \partial X_{L_1} \rightarrow \partial X_{L_2}$ maps the knot γ_K^1 to γ_K^2 for any knot K in S^3 , and that φ extends to a

FIGURE 17. X_L

homeomorphism $X_{L_1} \rightarrow X_{L_2}$ but cannot extend to any diffeomorphism $X_{L_1} \rightarrow X_{L_2}$. Then we call each X_{L_i} a *hook* and call $(X_{L_1}, X_{L_2}, \varphi)$ a *hook pair*. We note that any diffeomorphism between the boundaries of contractible smooth 4-manifolds extends to a homeomorphism between the contractible 4-manifolds according to [15], and that such a diffeomorphism necessarily preserves the framing of γ_K (This can be seen by comparing the intersection forms of 4-manifolds X_{L_i} with a 2-handle attached along γ_K^i). For a smooth 4-manifold Z containing X_{L_1} as a submanifold, remove X_{L_1} from Z and glue X_{L_2} via the gluing map φ . We call this operation a *hook surgery* along $(X_{L_1}, X_{L_2}, \varphi)$.

FIGURE 18. Knots γ_K^1 and γ_K^2 in ∂X_{L_1} and ∂X_{L_2}

By Theorem 3.2 and Lemma 3.1, (V_n, V_n^*, g_n^*) is a hook pair for each $n \geq 0$. Furthermore Corollary 3.3 tells that a hook surgery along (V_n, V_n^*, g_n^*) has the same operation as a cork twist along (V_n, g_n) . As seen from our results, a hook surgery is useful to obtain pairs of (non-concordant) knots and links representing exotic pairs of 4-manifolds. Hence, constructing links in S^3 admitting a hook pair is a natural problem. It is also interesting to characterize corks of Mazur type admitting compatible hook surgeries.

5.2. Dot-zero surgery. For an ordered link L of two unknotted components in S^3 , we denote the link L with the reversed order by \tilde{L} . Let X_L be the 4-dimensional handlebody obtained from L by putting a dot on the first component and a 0 on top of the second component (see Figure 17). By exchanging the dot and zero, we obtain the handlebody $X_{\tilde{L}}$. Since $X_{\tilde{L}}$ is obtained from X_L by surgering $S^1 \times D^3$ to $D^2 \times S^2$ and then surgering the other $D^2 \times S^2$ to $S^1 \times D^3$, this operation induces a diffeomorphism $\varphi_L : \partial X_L \rightarrow \partial X_{\tilde{L}}$. For a smooth 4-manifold Z containing X_L as a submanifold, remove X_L and glue $X_{\tilde{L}}$ back via the gluing map φ_L . We say that the resulting 4-manifold is obtained from Z by a *dot-zero surgery* along (X_L, φ_L) .

It is known that many important surgeries are essentially dot-zero surgeries along (often Stein) 4-manifolds. Logarithmic transform, Fintushel-Stern's rational blowdown, cork twists and plug twists are such examples (cf. [4, 6, 17]). Therefore, characterizing a link which can (or cannot) alter diffeomorphism types preserving homeomorphism types is a natural problem. The characterization is also helpful to find a diffeomorphism between complicated handlebodies.

We here give a sufficient condition on a link such that any dot-zero surgery induced from the link does not change the diffeomorphism type of a 4-manifold. For an ordered link L of two unknotted components in S^3 , we present L as in the left diagram of Figure 19 using a tangle T_L . Let L' be the associated link given by the right diagram.

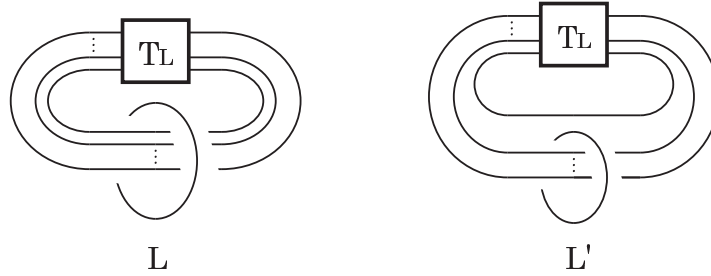


FIGURE 19. Link L and its associated link L' , where T_L is a tangle.

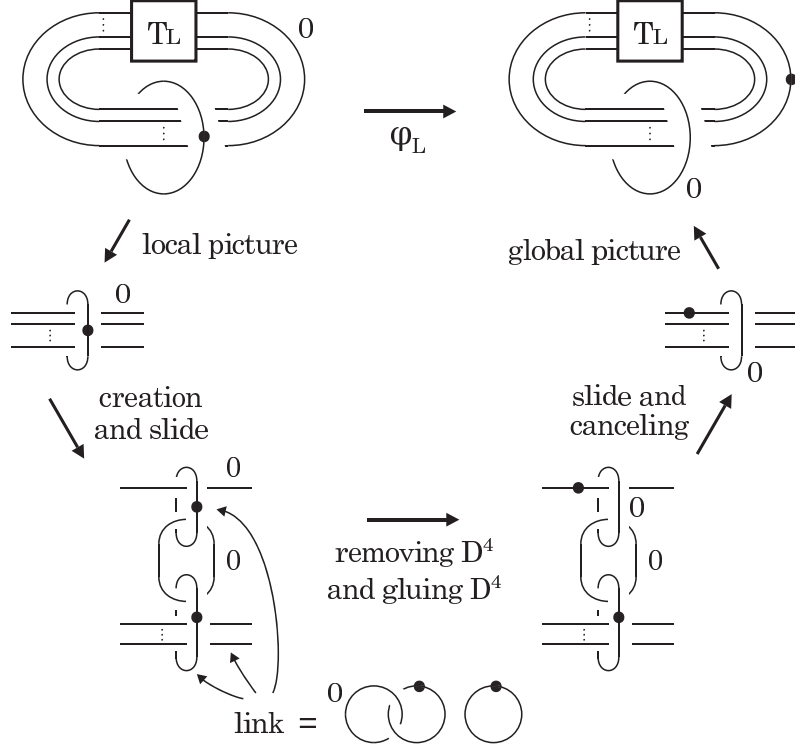
Proposition 5.1. *For an ordered link L of two unknotted components in S^3 , if the associated link L' is the trivial link, then the diffeomorphism $\varphi_L : \partial X_L \rightarrow \partial X_{\tilde{L}}$ extends to a diffeomorphism $X_L \rightarrow X_{\tilde{L}}$. Consequently, any dot-zero surgery along (X_L, φ_L) does not change the diffeomorphism type of a 4-manifold.*

Proof. The diffeomorphism $\varphi_L : \partial X_L \rightarrow \partial X_{\tilde{L}}$ is described in the upper side of Figure 20. The claim thus follows from the decomposition of φ_L in the figure and the fact that any self-diffeomorphism of S^3 extends to a self-diffeomorphism of D^4 . \square

Let L_n be the link in S^3 given by the diagram of V_n^* in Figure 4. Since $V_n^* (\cong V_n)$ admits a Stein structure for each $n \geq 0$, one might expect that a dot-zero surgery along (X_{L_n}, φ_{L_n}) can alter smooth structures of 4-manifolds. In fact, if a tangle T_L of a link L admits a Legendrian representative with $tb \geq 1$, then we can show that a dot-zero surgery along (X_L, φ_L) can alter smooth structures (cf. [5, 6]). However, by the above proposition, any dot-zero surgery along (X_{L_n}, φ_{L_n}) does not change smooth structures. Hence we obtain the following corollary.

Corollary 5.2. *For each integer n , the diffeomorphism $\varphi_{L_n} : \partial X_{L_n} \rightarrow \partial X_{\tilde{L}_n}$ extends to a diffeomorphism $X_{L_n} \rightarrow X_{\tilde{L}_n}$. Furthermore, X_{L_n} admits a Stein structure for $n \geq 0$.*

We remark that the Hopf link has been the only known such example to the best of the author's knowledge. Since $X_{L_n} = V_n^*$ is diffeomorphic to V_n , and V_n is a cork for each $n \geq 0$, any L_n ($n \geq 0$) is not isotopic to the Hopf link. Note that the 4-ball is not a cork. These links L_n 's are probably mutually non-isotopic links (e.g. calculate Casson invariants of $\partial X_{L_n} \cong \partial V_n$), but we do not pursue this point here.

FIGURE 20. Decomposition of the diffeomorphism $\varphi_L : \partial X_L \rightarrow \partial X_{\tilde{L}}$

Since (V_n, g_n) is a cork for $n \geq 0$, and V_n is diffeomorphic to V_n^* , we obtain the corollary below.

Corollary 5.3. *There exist two links J_1, J_2 of two unknotted components in S^3 satisfying the following conditions.*

- The 4-manifolds X_{J_1} and X_{J_2} are diffeomorphic to each other.
- $\varphi_{J_1} : \partial X_{J_1} \rightarrow \partial X_{\tilde{J}_1}$ extends to a diffeomorphism $X_{J_1} \rightarrow X_{\tilde{J}_1}$, but $\varphi_{J_2} : \partial X_{J_2} \rightarrow \partial X_{\tilde{J}_2}$ cannot extend to any diffeomorphism $X_{J_1} \rightarrow X_{\tilde{J}_2}$.

Therefore this corollary together with the proof of Theorem 4.1 tells that the choice of a link presentation of a fixed contractible 4-manifold does affect the diffeomorphism type of a 4-manifold obtained by a dot-zero surgery along the fixed 4-manifold.

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